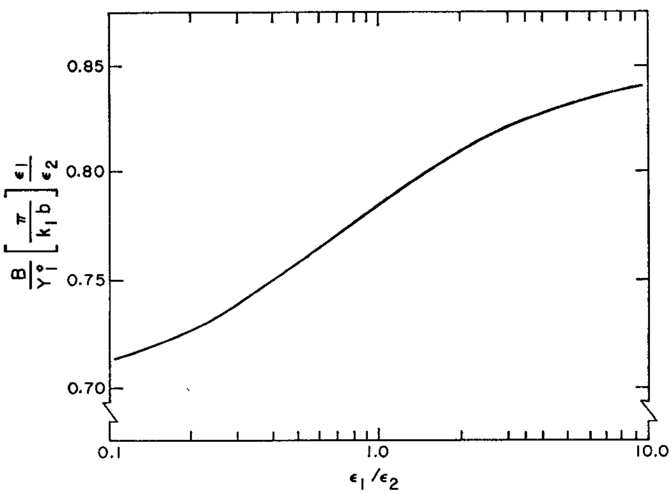

Fig. 2. Susceptance as a function of ϵ_1/ϵ_2.

and

$$P\int_{-1}^{1} \frac{X^{1-\beta}+X^{\beta}}{1-x} \ln \left| 1-\sqrt{1-x^2} \right| dx.$$

These integrals may be evaluated by changing the integration variable to X, replacing the logarithmic terms in the integrand by appropriate integrals involving dummy variables, and then reversing orders of integration. After some lengthy manipulations, one finds the following expression for the input admittance at $z=0$:

$$\frac{Y_{in}}{Y_1^0} = \frac{1}{2}\frac{Y_2^0}{Y_1^0} + j\frac{B}{Y_1^0} \tag{8}$$

where

$$\frac{B}{Y_1^0} = \frac{k_1 b}{\pi}\alpha^2 \frac{\epsilon_2}{\epsilon_1}\left[\frac{\pi}{2\alpha} - 2\ln 2 - \Psi(1-\beta) - \gamma\right] \tag{9}$$

where $\Psi(x)$ is Euler's psi-function and γ is Euler's constant.

III. Discussion of Results

In order to ascertain the validity of the solution, two convenient checks are available.

The first check is the edge condition. It is well known that the behavior of the field near a sharp corner plays an important role in the uniqueness of the solution. Using (7), one can show that near the corner at $z=0$, the electric field behaves as

$$E(\Delta) \sim 0(\Delta^{-(1-2\beta)}) \tag{10}$$

where $\Delta=(b-y)$. For equal media, $\beta=1/3$, and the field behaves as

$$E(\Delta) \sim 0(\Delta^{-1/3})$$

a well-known result. In general, we may use Mittra and Lee [2] and find that

$$E(\Delta) \sim 0(\Delta^{\tau-1}) \tag{11}$$

where

$$\tau - 1 = -\frac{1}{\pi}\tan^{-1}\left(\frac{1}{m}\sqrt{1+2m}\right)$$

and $m=\epsilon_1/\epsilon_2$. By using a trigonometric identity, it is seen that the results in (10) and (11) are identical.

Another check is a comparison with the known solution of the case $\epsilon_1=\epsilon_2$. Hence, $\beta=1/3$, and we have

$$\frac{B}{Y_1^0} = \frac{4b}{\lambda}\left[\frac{9}{4}\ln 3 - 3\ln 2\right]$$

which agrees with the predominant term of Marcuvitz [1].

Hence, (9) should be accurate to within a few percent for reasonable choices of material parameters.

Fig. 2 illustrates the behavior of $(B/Y_1^0)(\pi/k_1 b)(\epsilon_1/\epsilon_2)$ as a function of $m=\epsilon_1/\epsilon_2$. It is seen to be nearly constant over at least two decades of m. It may also be noted that, for the quasi-static approximation used here, the permeabilities do not appear in the expression for the discontinuity susceptance, which is purely capacitive.

IV. Conclusions

This short paper has found the effects of a 2:1 E-plane waveguide step with a simultaneous change of media. The solution has been found by application of the theory of singular integral equations. It should be noted that the conformal mapping solution is no longer valid because of the change in media. Other methods may be used to solve this problem for an arbitrary step ratio; however, these solutions do not exhibit the simplicity of (9). Perhaps in its simplicity, the solution obtained here can guide one in the effects of such a discontinuity. If desired, one might then proceed to a more complete solution.

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The Synthesis of Quarter-Wave Coupled Circulators with Chebyshev Characteristics

J. HELSZAJN

Abstract—The purpose of this short paper is to give an exact theory of quarter-wave coupled circulators with Chebyshev characteristics. The synthesis starts by replacing the lumped-element equivalent shunt resonator of the circulator by a distributed one that has the same susceptance slope parameter as the original circuit. In this way the overall network involves commensurate transmission lines only. The bandwidth over which the assumed form of the equivalent circuit applies is carefully discussed in terms of the two split frequencies of the magnetized junction. Tables for the required circulator parameters and transformer admittances for one and two transformer sections as a function of VSWR and bandwidth are included. The realizable solution for the latter arrangement is severely restricted by the equivalent circuit of the basic junction. Experimental results on an octave-band stripline circulator, with a two-section transformer, are also included.

I. Introduction

An important property of the 3-port junction circulator is that an ideal circulator is obtained when the junction is matched [1]. One well-known method of broad-banding this device is to use external matching networks. One arrangement that is often used consists of a cascade of quarter-wave transformers [2]–[4].

An approximate theory for this type of network has been given in [2]. The purpose of this short paper is to give an exact synthesis procedure for the case of one- and two-step transformers that will give an equal-ripple Chebyshev response for the reflection coefficient of the overall circulator network. This short paper assumes in the usual way that the equivalent network at the reference terminals of the junction consists of a shunt lumped-element resonator in parallel with the gyrator conductance of the circulator [4]–[7]. The synthesis procedure then starts by replacing the lumped-element resonator by a distributed one consisting of a quarter-wave short-circuited transmission line that has the same susceptance slope parameter as the original circuit. The two circuits are equivalent provided their susceptance slope parameters are the same. The equivalent circuit of the device can therefore be represented by a quarter-wave short-circuited transmission line in parallel with the gyrator conductance of the circulator. The admittance of the distributed network is uniquely related to the susceptance slope parameter, once the nature of the net-

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work is stated. The equivalent circuit of the complete network now involves commensurate quarter-wave transmission lines only. It can be made to have a Chebyshev response by forming the reflection coefficient of the circuit and equating its zeros and maxima to give a characteristic that is a Chebyshev polynomial of the first kind. The results have been obtained with the help of a computer program, and are presented in tabular form for $n=2$ and $n=3$ in terms of the necessary bandwidth and VSWR of the overall network. These tables give the susceptance slope parameter, the gyrator conductance, and the loaded Q factor of the circulator network. They also give the admittance values of the matching transformers.

The bandwidth over which the assumed form for the circulator equivalent circuit applies is carefully discussed in terms of the two split frequencies of the magnetized junction. The assumption about the equivalent circuit is satisfied provided the overall bandwidth of the circulator is equal to or less than the bandwidth between the two split frequencies. This puts one constrain on the realizable frequency response of the overall junction.

Physically realizable values that the susceptance slope parameter and gyrator conductance of practical circulators can take are also discussed. The susceptance slope parameter is primarily determined by the geometry of the device and the gyrator conductance by the magnetization and the direct magnetic field. The ratio of these two quantities defines the loaded Q factor of the junction. It is related to the split frequencies of the magnetized junction and to the magnetic variables of the junction [2]–[5]. This factor places a further constraint on the frequency response of the overall network.

The permissible values of these variables are thus limited to a much smaller range than depicted in the tables. This is particularly true for $n=3$, where the range of realizable solutions is severely restricted. However, such an $n=3$ circuit is suitable for the construction of the octave-band circulator. The $n=2$ network is a much more versatile one in that it allows a wide range of ripple levels and frequency responses to be obtained. Some entries inappropriate to the design of circulators have been included for completeness because of their potential applicability to other circuit problems.

It is also observed that a more relaxed requirement on the loaded Q factor is possible provided the reflection coefficient passes through a minimum value instead of through zero [8], [9]. This short paper also includes experimental results on an octave-band stripline circulator, using an $n=3$ network.

II. The Equivalent Circuit of the Junction Circulator

When the junction operates as a circulator, it is always possible to represent it by an equivalent lumped shunt resonator at a pair of terminals, which is usually taken at the edge of the ferrite disks. This is shown in Fig. 1. The elements of this resonator may be obtained experimentally from the admittance function. The bandwidth of this network is determined in the normal way by its loaded Q factor, which is given by

$$Q_L = \frac{b'}{g} = \frac{\text{normalized susceptance slope parameter}}{\text{normalized shunt conductance}}. \quad (1)$$

It is the purpose of this short paper to obtain the element values for b', g, and Q_L for the case of quarter-wave coupled circulators with $n=2$ and 3.

Another form for the loaded Q-factor Q_L of circulators, which rely on 30° splitting between the degenerate modes, is [3]–[5]

$$\frac{1}{Q_L} = \sqrt{3}\left(\frac{\omega_{+1}-\omega_{-1}}{\omega_0}\right) \quad (2)$$

where ω_{+1} and ω_{-1} are the frequencies of the two split modes of the magnetized junction, and ω_0 is the center frequency.

Combining these two equations gives

$$g = \sqrt{3}\, b'\left(\frac{\omega_{+1}-\omega_{-1}}{\omega_0}\right). \quad (3)$$

Equation (3) may be taken as a universal definition for the input admittance g at the center frequency of junction circulators that rely on 30° splitting between the degenerate modes. One important property of this circuit is that the susceptance slope parameter is essentially determined by the geometry of the junction and remains nearly constant over fairly large variations in the shunt conductance. It is observed that this variable contains the frequency behavior of the junction. It is derived in [11] for a number of different geometries.

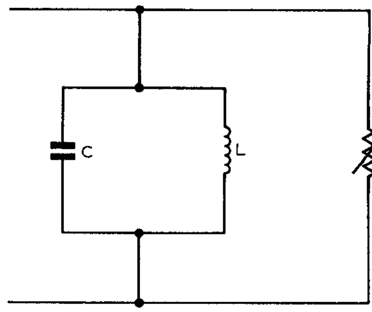

Fig 1. Lumped-element equivalent resonator circuit of circulator.

A second important property of this resonator is that its shunt conductance can be adjusted by removing the degeneracy between the two split modes of the magnetized junction. The maximum value of the gyrator admittance of quarter-wave coupled circulators is constrained by the external network, because it is difficult to realize admittance levels for the transformers in excess of about $y_t=5.5$.

The difference between the two split frequencies may be obtained by using perturbation theory. For most junctions it is related to the magnetic splitting K/μ by

$$\frac{\omega_{+1}-\omega_{-1}}{\omega_0} \approx \frac{K}{\mu} \quad (4)$$

where μ and K are the diagonal and off-diagonal components of the permeability tensor.

The ratio K/μ usually lies between 0 and 0.75, and it is this factor that determines the difference between the two split frequencies and the allowable range of Q_L. Using this criterion the difference between the normalized split frequencies lies between

$$0 < 2\delta_{0,+1} < 0.75 \quad (5)$$

and the loaded Q factor lies between

$$0.77 > Q_L > \infty. \quad (6)$$

The above results show that the minimum value of $Q_L=b'/g$ is a constant determined by the magnetic variables of the ferrite material.

The form of the equivalent circuit used in this short paper for the circulator is assumed to be reliable between the two split frequencies of the magnetized junction [3], [4]. The assumption about the equivalent circuit is satisfied provided the normalized bandwidth $2\delta_0$ is approximately equal to or less than $2\delta_{0,+1}$. Reasonable agreement between theory and experiment is therefore expected provided

$$2\delta_0 \le 2\delta_{0,+1}. \quad (7)$$

The relation between the required bandwidth parameter $2\delta_0$ and the split frequencies $2\delta_{+1}$ is therefore included in this short paper. Combining (5) and (7) shows that the maximum bandwidth of quarter-wave coupled circulators is

$$2\delta_0 = 0.75. \quad (8)$$

A simple method by which the difference between these two split frequencies can be obtained experimentally has been described elsewhere [12]. Fig. 2 shows one example obtained at 2 GHz. Here $2\delta_{0,+1}=0.54$.

Mathematical solutions for quarter-wave coupled circulators for which either $Q_L<0.77$, $2\delta_0>2\delta_{0,+1}$, or $y_t>5.5$ do not therefore lead to physically realizable devices.

III. Quarter-Wave Coupled Circulator with Chebyshev Frequency Response

One well-known method of overcoming the physical limitations of the ferrite parameters is to use external matching transformers to improve the bandwidth. The purpose of this short paper is to obtain the values of b' and g and the external parameters for which the overall response has an equal-ripple Chebyshev characteristic.

The synthesis procedure starts by replacing the lumped-element resonator shown in Fig. 1 by a distributed one consisting of a quarter-wave short-circuited transmission line that has the same susceptance slope parameter as the original circuit. The equivalent circuit of the circulator has now the form shown in Fig. 3. The complete network

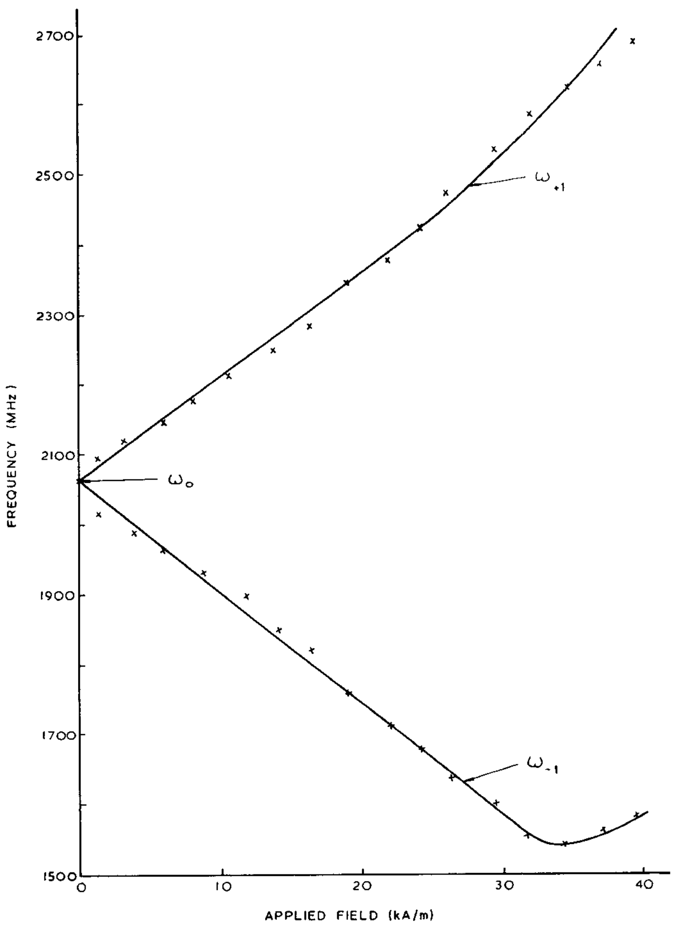

Fig. 2. Experimental split frequencies as a function of direct magnetic field for 2-GHz stripline circulator.

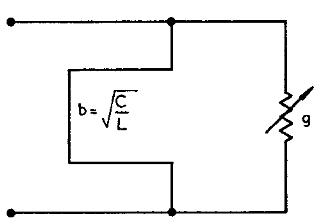

Fig. 3. Distributed element equivalent resonator circuit of circulator.

now involves commensurate quarter-wave transmission lines only for which an exact synthesis procedure can be developed.

The input admittance of the circulator y_L is

$$y_L = \frac{Y_L}{Y_0} = g - jb \cot \theta. \tag{9}$$

This equation may be compared with the one given by Bosma [14].

The frequency variable θ is

$$\theta = \frac{\pi}{2}(1 + \delta) \tag{10}$$

where

$$2\delta = 2\left(\frac{\omega_0 - \omega}{\omega_0}\right). \tag{11}$$

At the band edges one has

$$2\delta_0 = 2\left(\frac{\omega_0 - \omega_1}{\omega_0}\right). \tag{12}$$

Here ω_0 is the center frequency, ω is the frequency variable, and $\omega_{1,2}$ are the band edges. The normalized susceptance slope parameter b' is related to the normalized characteristic admittance b of the network by

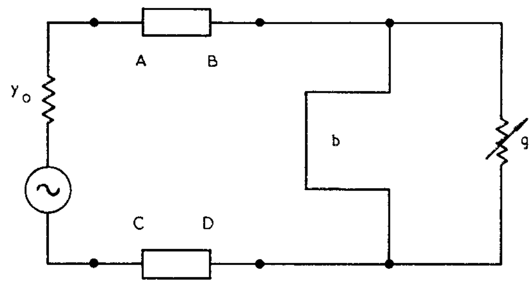

Fig. 4. Equivalent network in terms of overall $ABCD$ matrix.

$$b' = \frac{\pi}{4} b. \tag{13}$$

The equivalent network for matching each of the three ports of the circulator is shown in Fig. 4 in terms of the $ABCD$ matrix of the transformer. By straightforward calculations the admittance of the network at the input terminals is given in terms of the overall normalized $ABCD$ matrix of the circuit by

$$y_{in} = \frac{Y_{in}}{Y_0} = \frac{jC + Dy_L}{A + jBy_L} \tag{14}$$

where A, B, C, and D are real quantities.

In terms of the original variables the magnitude of the reflection coefficient is

$$|\Gamma| = (\Gamma\Gamma^*)^{1/2} = \frac{[(Dg - A - Bb \cot \theta)^2 + (C - Bg - Db \cot \theta)^2]^{1/2}}{[(Dg + A + Bb \cot \theta)^2 + (C + Bg - Db \cot \theta)^2]^{1/2}}. \tag{15}$$

The design proceeds by having the frequency at which the reflection coefficient passes through its zeros and maxima coincide with those of the Chebyshev polynomial. When the reflection coefficient passes through zero,

$$g = \frac{AD + BC}{B^2 + D^2} \tag{16}$$

$$b \cot \theta = \frac{CD - AB}{B^2 + D^2}. \tag{17}$$

The reflection coefficient passes through a maximum when

$$|\Gamma| = \gamma. \tag{18}$$

In terms of the maximum VSWR, γ is given by

$$\gamma = \frac{(r - 1)}{(r + 1)}. \tag{19}$$

IV. Synthesis of $n = 2$ Network

For a single-step transformer the elements of the normalized $ABCD$ matrix are

$$A = \cos \theta$$
$$B = \frac{\sin \theta}{y_{01}}$$
$$C = y_{01} \sin \theta$$
$$D = \cos \theta. \tag{20}$$

For $n = 2$ the Chebyshev polynomial passes through zero when

$$2x^2 - 1 = 0. \tag{21}$$

This occurs when the frequency variable is

$$x = \pm \frac{1}{\sqrt{2}}. \tag{22}$$

The Chebyshev polynomial passes through a maximum when

$$2x^2 - 1 = \pm 1. \tag{23}$$

This occurs when the frequency variable is

TABLE I

NETWORK PARAMETERS FOR $n=2$

$2\delta_0$	b'	g	θ	Y_1
R=1.05				
0.100	50.304	16.424	3.063	4.153
0.150	15.433	7.846	1.967	2.870
0.200	6.809	4.844	1.406	2.255
0.250	3.672	3.455	1.063	1.905
0.300	2.248	2.700	0.833	1.684
0.350	1.501	2.245	0.669	1.535
0.400	1.067	1.950	0.547	1.431
0.450	0.795	1.747	0.455	1.355
0.500	0.614	1.603	0.383	1.297
0.550	0.488	1.496	0.326	1.253
0.600	0.397	1.414	0.280	1.219
0.660	0.317	1.340	0.237	1.186
R=1.10				
0.100	133.831	30.445	4.396	5.787
0.150	40.384	14.070	2.870	3.934
0.200	17.457	8.339	2.093	3.029
0.250	9.205	5.686	1.619	2.501
0.300	5.508	4.245	1.297	2.161
0.350	3.598	3.377	1.065	1.927
0.400	2.505	2.813	0.890	1.759
0.450	1.831	2.427	0.754	1.634
0.500	1.390	2.151	0.646	1.538
0.550	1.088	1.946	0.559	1.463
0.600	0.872	1.791	0.487	1.404
0.660	0.687	1.650	0.417	1.347
R=1.15				
0.100	234.000	43.247	5.411	7.052
0.150	70.196	19.753	3.554	4.766
0.200	30.113	11.530	2.612	3.641
0.250	15.738	7.724	2.038	2.980
0.300	9.328	5.656	1.649	2.550
0.350	6.032	4.410	1.368	2.252
0.400	4.158	3.601	1.155	2.035
0.450	3.010	3.047	0.988	1.872
0.500	2.264	2.651	0.854	1.746
0.550	1.756	2.358	0.745	1.647
0.600	1.397	2.135	0.654	1.567
0.660	1.091	1.932	0.565	1.491
R=1.20				
0.100	344.387	54.983	6.264	8.123
0.150	102.999	24.962	4.126	5.473
0.200	44.010	14.454	3.045	4.165
0.250	22.892	9.591	2.387	3.393
0.300	13.495	6.950	1.942	2.886
0.350	8.677	5.357	1.620	2.536
0.400	5.947	4.324	1.375	2.278
0.450	4.279	3.616	1.184	2.083
0.500	3.200	3.109	1.029	1.932
0.550	2.469	2.735	0.903	1.812
0.600	1.954	2.451	0.797	1.715
0.660	1.517	2.191	0.692	1.622

$$x = 0 \tag{24}$$

and

$$x = \pm 1. \tag{25}$$

The Chebyshev polynomials used here are discussed in [8] and [13]. The transformer admittance y_{01}, the normalized characteristic admittance b, and the normalized shunt conductance g are now obtained by adjusting the zeros and maxima of the reflection coefficient to coincide with those of the Chebyshev polynomial. This yields three equations from which y_{01}, b, and g are obtained.

At $x=0$, (18) applies and one obtains

$$y_{01}^2 = rg \tag{26}$$

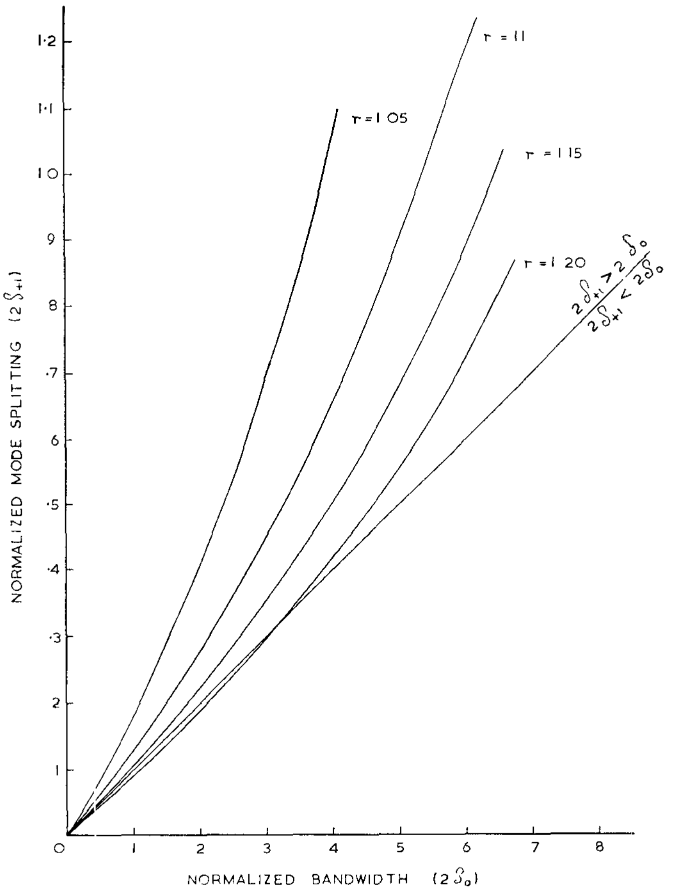

Fig. 5. Relation between $2\delta_0$ and $2\delta_{+1}$ for $n=2$ and parametric values of r.

when (18) is evaluated with

$$\cos\theta = 0. \tag{27}$$

At $x=\pm(1/\sqrt{2})$ the reflection coefficient $|\Gamma|=0$. Here, the two independent equations given by (16) and (17) apply. The result for b and g in terms of the original variables is

$$g = \frac{r - \sin^2\theta}{r\cos^2\theta} \tag{28}$$

and

$$b = \sqrt{\frac{g}{r}}\,(rg - 1)\sin^2\theta \tag{29}$$

which must be evaluated with

$$\cos\theta = \frac{1}{\sqrt{2}}\cos\theta_0. \tag{30}$$

g and b' are given in tabular form as a function of r and $2\delta_0$ with the help of a computer program in Table I. Fig. 5 shows the relation between $2\delta_0$ and $2\delta_{0,+1}$ for this type of network. In addition to this constrain the entries in these tables for which either $Q_L<0.77$ or $y_{01}>5.5$ are also not suitable for the construction of quarter-wave coupled circulators. The permissible solutions for the $n=2$ network are boxed in Table I. It shows, among other results, that a junction with a constant susceptance slope parameter can be used to construct circulators with different ripple levels and frequency responses. This makes this type of junction very versatile indeed. This is in good agreement with practical experience.

V. SYNTHESIS OF $n=3$ NETWORK

For a double-step transformer the elements of the normalized $ABCD$ matrix are

$$A = \cos^2\theta - \frac{y_{02}}{y_{01}}\sin^2\theta$$

TABLE II
NETWORK PARAMETERS FOR $n=3$

R=1·05

$2\delta_0$	b'	θ	g	Y1	Y2
0,100	20633,204	4.830	4271,997	6.245	408.185
0,150	2724.182	3.193	853,154	4.210	122.959
0,200	649.105	2.366	274,365	3.206	53.101
0,250	213,973	1.862	114,918	2.615	28,029
0,300	86.710	1.519	57,069	2.230	16.644
0,350	40.567	1.269	31,973	1.963	11,097
0,400	21.108	1.076	19,624	1.769	7.836
0,450	11.925	0.921	12,948	1.624	5.844
0,500	7,196	0.794	9,066	1.513	4,556
0,550	4,583	0.687	6,674	1.427	3.686
0,600	3,054	0.596	5,127	1.358	3.076
0,660	1,973	0.503	3,921	1.294	2.563

R=1·10

$2\delta_0$	b'	θ	g	Y1	Y2
0,100	60931.440	6.245	9757,171	8.032	793.430
0,150	8025.626	4.138	1939,352	5.391	237.430
0,200	1905.491	3.077	619,236	4.082	101,578
0,250	625.015	2.434	256,800	3.305	52,965
0,300	251.612	1.999	125,856	2.795	31.351
0,350	116,728	1.684	69,333	2.436	20,285
0,400	60.108	1.442	41,677	2.173	14,028
0,450	33.539	1.250	26,824	1.973	10,218
0,500	19.949	1.093	18,249	1.817	7.763
0,550	12.502	0.961	13,004	1.694	6.108
0,600	8,184	0.849	9,641	1.594	4.950
0,660	5.166	0.734	7,043	1.499	3.977

R=1·15

$2\delta_0$	b'	θ	g	Y1	Y2
0,100	112327.120	7.290	15407,534	9.358	1161,559
0,150	14781.587	4.836	3056,774	6.270	346.678
0,200	3504.736	3.601	973,366	4.736	147.766
0,250	1147.428	2.853	402,149	3.824	76.676
0,300	460.787	2.349	196,125	3.221	45.115
0,350	213,102	1.985	107,371	2.797	28.982
0,400	109.314	1.707	64,048	2.483	19,875
0,450	60.713	1.487	40,841	2.244	14.340
0,500	35.914	1.307	27,482	2.056	10,779
0,550	22,365	1.157	19,338	1.906	8.382
0,600	14,537	1.028	14,135	1.784	6,707
0,660	9.090	0.897	10,131	1.666	5,302

R=1·20

$2\delta_0$	b'	θ	g	Y1	Y2
0,100	171035.731	8.157	20967,327	10.456	1514.381
0,150	22495.848	5.413	4155,581	7.001	451.322
0,200	5329.825	4.034	1321,288	5.281	191.973
0,250	1743.184	3.200	544,790	4.256	99.347
0,300	699.110	2.638	264,990	3.579	58.261
0,350	322,783	2.232	144,591	3.100	37.279
0,400	165.238	1.924	85,897	2.746	25.446
0,450	91.547	1.680	54,503	2.474	18.262
0,500	53.996	1.481	36,462	2.260	13,645
0,550	33.512	1.315	25,483	2.088	10,540
0,600	21.698	1.174	18,483	1.948	8.374
0,660	13.497	1.030	13,109	1.811	6.557

$$B = \left(\frac{1}{y_{01}} + \frac{1}{y_{02}}\right) \sin\theta\cos\theta$$
$$C = (y_{01} + y_{02}) \sin\theta\cos\theta$$
$$D = -\frac{y_{01}}{y_{02}} \sin^2\theta + \cos^2\theta. \tag{31}$$

For $n=3$ the Chebyshev polynomial passes through zero when

$$4x^3 - 3x = 0. \tag{32}$$

This occurs when the frequency variable takes the values

$$x = 0 \tag{33}$$

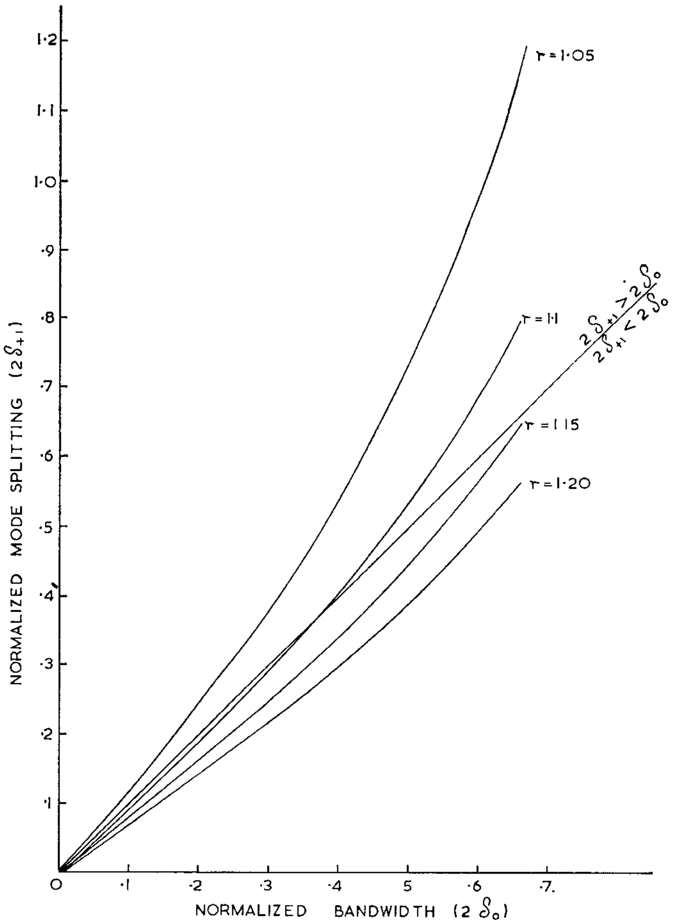

Fig. 6. Relation between $2\delta_0$ and $2\delta_{+1}$ for $n=3$ and parametric values of r.

and

$$x = \pm\sqrt{\tfrac{3}{4}}. \tag{34}$$

The Chebyshev polynomial passes through a maxima when

$$4x^3 - 3x = \pm 1. \tag{35}$$

This occurs when the frequency variable takes the values

$$x = \pm\tfrac{1}{2} \tag{36}$$

and

$$x = \pm 1. \tag{37}$$

The admittances of the two transformers y_{01} and y_{02}, the normalized characteristic admittance b, and the shunt conductance g are now obtained by adjusting the zeros and maxima of the reflection coefficient to coincide with those of the Chebyshev polynomial. This gives four equations from which y_{01}, y_{02}, b, and g are obtained.

At $x=0$ the reflection coefficient $|\Gamma|$ is zero. Here, (16) and (17) apply. This condition yields

$$y_{01}^2 = \frac{y_{02}^2}{g} \tag{38}$$

when (16) is evaluated with

$$\cos\theta = 0. \tag{39}$$

At $x = \pm\sqrt{\frac{3}{4}}$ the reflection coefficient is also zero, so that (16) and (17) apply again. From (16) one obtains

$$y_{02}^2 = \frac{g(1+\sqrt{g})^2 \sin^2\theta\cos^2\theta}{1 - (\sqrt{g}\cos^2\theta - \sin^2\theta)^2} \tag{40}$$

which must be evaluated with

$$\cos\theta = \sqrt{\tfrac{3}{4}} \cos\theta_0. \tag{41}$$

TABLE III
OCTAVE-BAND $n=3$ NETWORKS

r	$2\delta_0$	$2\delta_{0,+1}$	b'	g	Q_L	Y_1	Y_2
1.11	0.660	0.750	5.903	7.667	0.770	1.534	4.249
1.12	0.660	0.719	6.665	8.288	0.804	1.569	4.517
1.13	0.660	0.690	7.452	8.906	0.837	1.602	4.781
1.14	0.660	0.666	8.261	9.521	0.868	1.634	5.043

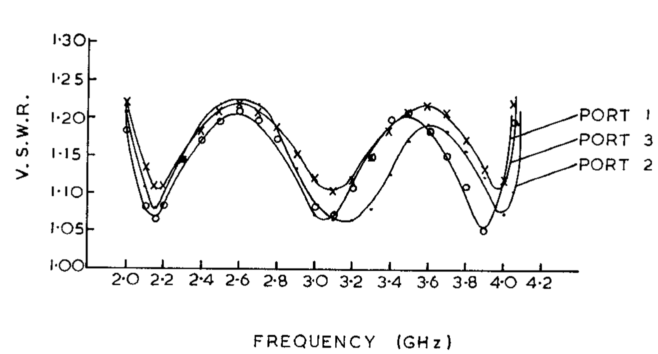

Fig. 7. Frequency response of 2–4-GHz octave-band circulator.

g and b are now obtained by forming two additional equations. One of these is (17), which applies when $|\Gamma|=0$ at $x=\pm\sqrt{\frac{3}{4}}$. The other is obtained at $x=\pm\frac{1}{2}$ when $|\Gamma|=\gamma$. Here (18) applies. The result is

$$b \cot\theta = \frac{CD-AB}{B^2+D^2} \quad (42)$$

which must be evaluated at

$$\cos\theta = \sqrt{\tfrac{3}{4}}\cos\theta_0$$

and

$$\gamma\,[(Dg+A+Bb\cot\theta)^2+(C+Bg-Db\cot\theta)^2]^{1/2} = [(Dg-A-Bb\cot\theta)^2+(C-Bg-Db\cot\theta)^2]^{1/2} \quad (43)$$

which must be evaluated at

$$\cos\theta = \tfrac{1}{2}\cos\theta_0. \quad (44)$$

y_{01}, y_{02}, g, and b' are given in tabular form as a function of r and $2\delta_0$ with the help of a computer program in Table II. These tables indicate the precision with which circulator parameters must be satisfied in order to obtain a specified Chebyshev response. Fig. 6 shows the relation between $2\delta_0$ and $2\delta_{0,+1}$ for this type of network. The useful entries for this arrangement are constrained by (3) and (4). Applying the constraint $Q_L>0.77$ and $y_{02}<5.5$ to these tables shows that the freedom to specify the overall performance with an $n=3$ quarter-wave coupled circulator is severely curtailed with this arrangement. The permissible solutions are boxed in Table II.

With $2\delta_0=0.66$ and $n=3$ the realizable solutions are completely constrained by the equivalent circuit of the junction to $r=1.11$, 1.12, 1.13, and 1.14. The values for the susceptance slope parameter are, for these solutions, particularly simple to realize.

VI. Experimental Octave-Band Circulator

This section describes the construction of an $n=3$ octave-band stripline circulator in the frequency range 2–4 GHz. The permissible solutions for such a circulator are restricted to the ones given in Table III. The normalized magnetization for the garnet material used in this circulator was about 0.72. This coincides with a minimum loaded Q factor of 0.80. The nominal values for the transformer admittances y_{01} and y_{02} were 1.43 and 3.95. The measured susceptance slope parameter for this junction in a directly coupled configuration was 6.4. Fig. 7 shows the experimental results obtained on such 2–4-GHz circulator. This result was obtained by trimming each transformer with the help of a tuning screw. Although it is not possible to state exactly the final values used for y_{01} and y_{02} this tuning arrangement increases the admittances y_{01} and y_{02} from the nominal values as given above to the ones stated in Table II. The insertion loss for this circulator was below 0.50 dB. The maximum VSWR was typically 1.22 and the minimum one was 1.07. Such a result is consistent with a slightly more relaxed requirement on the loaded Q factor of the junction than stated in the theoretical results given here.

VII. Conclusions

This short paper has developed the exact solution of the quarter-wave coupled circulator for $n=2$ and $n=3$. The results are presented in tabular form from which the junction parameters and details of the transformer sections may be obtained. These tables indicate the precision with which circulator parameters must be satisfied in order to obtain a prescribed Chebyshev response. The physical constrains on the entries of these tables have also been discussed. The experimental results include an $n=3$ octave-band circulator.

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